

Entropic force approach in a noncommutative charged black hole and the equivalence principle

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Abstract

Recently, Verlinde has suggested a novel model of duality between thermodynamics and gravity which leads to an emergent phenomenon for the origin of gravity and general relativity. In this paper, we investigate some features of this model in the presence of noncommutative charged black hole by performing the method of coordinate coherent states representing smeared structures. We derive several quantities, e.g. temperature, energy and entropic force. Our approach clearly exhibits that the entropic force on a smallest fundamental cell of holographic surface with radius r_0 is halted. Accordingly, we can conclude that the black hole remnants are absolutely inert without gravitational interactions. So, the equivalence principle of general relativity is contravened due to the fact that it is now possible to find a difference between the gravitational and inertial mass. In other words, the gravitational mass in the remnant size does not emit any gravitational field, therefore it is experienced to be zero, contrary to the inertial mass. This phenomenon illustrates a good example for a feasible experimental confirmation to the entropic picture of Newton's Second law in very short distances.

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The thermal emission from black holes has primarily proved the firm relations between thermodynamics and the gravitational treatment of black holes [1]. It seems that one noticeable evidence for the essence of gravity is perceived from the precise inspection of black holes thermodynamics because it is feasible to prepare a physical similarity between spacetimes comprising horizons and the concepts of temperature or entropy. Moreover, since a quantum theory of gravity tells us that the black hole entropy could be connected to a number of microscopic states, the meticulous assessment of black hole entropy, due to possessing the significant concepts for a yet to be formulated theory of quantum gravity, can be important to construct a full theory of quantum gravity. This is the principal reason to investigate the origin of the black hole entropy at a fundamental level. A few years ago, Jacobson demonstrated that the Einstein equations are attained from the thermodynamics laws [2]. In 2009, Padmanabhan performed the debate of equipartition energy of horizons to provide a thermodynamic outlook on gravity [3]. In 2010, Verlinde proposed a new viewpoint of duality between thermodynamics and gravity which has emerged, *as an entropic force*, by information changes linked to the locations of physical objects [4]. This hypothesis indicates that gravitational interaction emerges from the statistical treatment of microscopic degrees of freedom encrypted on a holographic surface and can be illustrated as a type of entropic force associated with the information which is accumulated on the holographic screens. The concept of entropic force in various situations has been scrutinized by many authors [5]. In addition, there are some comments on Verlinde's entropic gravity approach which point out short comings of this approach as well as open challenges [6], they provide some unanswered questions about the issue.

Considering the gravitational force is entropic-like and entropy combines the emergent view of gravity with the fundamental microstructure of a quantum spacetime, it is necessary to take into consideration the microscopic scale influences by applying precise implements like noncommutative gravity to illustrate the microscopic structure of a quantum spacetime, in Verlinde's proposal. In 2005, Nicolini *et al* [7] in a novel method to noncommutative gravity, which is established upon coordinate coherent state approach [8], recuperated the short distance treatment of point-like structures. This method is the so-called *noncommutative geometry inspired model*. They have clarified that the evaporation procedure of black hole must be ceased when the black hole gets to a minimal nonzero mass called a stable Planck-sized remnant. This residual mass of the black hole arises from the existence of minimal

observable length. The smallness of this scale would infer that noncommutativity effects can be apprehended just in extreme energy phenomena. The majority of the phenomenological analysis of the noncommutativity scenarios have assumed that the noncommutative energy scale cannot lie far above the TeV scale [9]. On the other hand, the fundamental Planck scale in models with extradimensions [10] can be adjacent to current particle physics experiments [11]; it is feasible to set the noncommutativity effects in a TeV regime. In addition, some type of divergencies which are revealed in general relativity can be removed in the noncommutativity framework. To illustrate more features, see [12] and the references included.

In this paper, we use noncommutative geometry inspired model to unite the microscopic structure of spacetime with the entropic interpretation of gravity due to the fact that the idea of entropy has a substantial connection to the quantum spacetime structures. We investigate the entropic force method in the presence of noncommutative Reissner-Nordström (RN) black hole to observe the possible novel phenomena due to the effects of smearing of the particle mass and charge. Our studies declare that there is a contravention of the equivalence principle of general relativity when we include the noncommutativity corrections in our calculations. The equivalence principle (EP) is a perceivable foundation for general relativity claiming that one cannot find a difference between a uniform acceleration and a gravitational field in a locally frame of reference. We will show that an evident violation of the EP exists. This enables one to locally mark a difference between the gravitational and inertial mass. Thus, an inherent trait for the fundamental microstructure of a quantum spacetime like noncommutative gravity can lead to a violation of the EP. In a recent paper [13], we addressed several issues of entropic nature of gravity as proposed by Verlinde in the framework of noncommutative geometry inspired model for the Schwarzschild black hole. As a result of spacetime noncommutativity, Einstein equations in vacuum have a Schwarzschild black hole solution which has a mass distributed in a region in place of a mass completely localized at a point. We implemented two different distributions: (a) Gaussian and (b) Lorentzian, in order to derive several quantities, e.g. temperature, energy and entropic force. Both mass distributions prepared the similar quantitative aspects for the entropic force. In this setup, if one considered the screen radius less than the radius of smallest holographic surface, one would encounter some unusual dynamical features leading to negative entropic force, i.e. gravitational repulsive force and it is worth of mentioning: at this regime either

our analysis is not the proper one, or non-extensive statistics should be employed. Therefore, we will henceforth apply the circumstance that the screen radius is bigger than the radius of the smallest holographic surface.

In the end, it should be noted that there have been other proposals which contravene the EP such as the quantum phenomenon of neutrino oscillations [14], comparing Hawking radiation to Unruh radiation [15], and examination of entropic picture of Newton's second law for the case of circular motion [16] (somewhat more related to the present work).

The method we consider here is to search for a static, asymptotically flat, spherically symmetric, minimal width, Gaussian distribution of mass and charge whose noncommutative size is made by the parameter $\sqrt{\theta}$. For this purpose, we are going to exhibit the mass and charge distributions by a smeared delta function ρ (see [7, 12, 17–19])

$$\begin{cases} \rho_m(r) = \frac{M}{(4\pi\theta)^{\frac{3}{2}}} e^{-\frac{r^2}{4\theta}} \\ \rho_e(r) = \frac{Q}{(4\pi\theta)^{\frac{3}{2}}} e^{-\frac{r^2}{4\theta}}, \end{cases} \quad (1)$$

where θ is the smallest fundamental cell of observable area in the noncommutative coordinates, beyond which coordinate resolution is vague. The solution of the Einstein equations associated with the above smeared sources leads to the metric of noncommutative RN black hole as follows [24]

$$ds^2 = -\left(1 - \frac{2M_\theta}{r} + \frac{Q_\theta^2}{r^2}\right) dt^2 + \left(1 - \frac{2M_\theta}{r} + \frac{Q_\theta^2}{r^2}\right)^{-1} dr^2 + r^2 d\Omega^2, \quad (2)$$

where $d\Omega^2 = d\vartheta^2 + \sin^2\vartheta d\varphi^2$ is the line element on the 2-dimensional unit sphere. The smeared mass and charge distributions are respectively given by

$$\begin{cases} M_\theta = M \left[\mathcal{E}\left(\frac{r}{2\sqrt{\theta}}\right) - \frac{r}{\sqrt{\pi\theta}} e^{-\frac{r^2}{4\theta}} \right] \\ Q_\theta = Q \sqrt{\mathcal{E}^2\left(\frac{r}{2\sqrt{\theta}}\right) - \frac{r}{\sqrt{2\pi\theta}} \mathcal{E}\left(\frac{r}{\sqrt{2\theta}}\right)}. \end{cases} \quad (3)$$

In the limit of $\frac{r}{\sqrt{\theta}} \rightarrow \infty$, the Gaussian error function defined as $\mathcal{E}(x) \equiv \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$, tends to one and we recover the ordinary mass and charge perfectly localized at a point, i.e. $M_\theta \rightarrow M$ and $Q_\theta \rightarrow Q$. Now, using the Killing equation

$$\partial_a \xi_b + \partial_b \xi_a - 2\Gamma_{ab}^c \xi_c = 0, \quad (4)$$

where $a, b, c \in \{0, 1, 2, 3\}$. And the condition of static spherical symmetry $\partial_0 \xi_a = \partial_3 \xi_a = 0$, and also the infinity condition $\xi_a \xi^a = -1$, the timelike Killing vector of the noncommutative

charged black hole can be written as

$$\xi_a = \left(\frac{2M_\theta}{r} - \frac{Q_\theta^2}{r^2} - 1, 0, 0, 0 \right), \quad (5)$$

which equals to zero at the event horizon. To illustrate a foliation of space, and perceiving the holographic screens Ω at surfaces of constant redshift, we consider the potential ϕ as follows

$$\phi = \frac{1}{2} \log(-\xi^a \xi_a), \quad (6)$$

we notice that e^ϕ implies the redshift factor and exhibits a link between the local time coordinate and the reference point with $\phi = 0$ at infinity.

The acceleration a^a on the spherical holographic screen with radius r , for a particle which is situated extremely neighboring the screen, is found to be [4]

$$a^a = -\nabla^a \phi. \quad (7)$$

It is clear that the acceleration is perpendicular to the holographic screen. By representing the normal vector as $N^a = \frac{\nabla^a \phi}{\sqrt{\nabla^b \phi \nabla_b \phi}}$, the local temperature on the screen is given by

$$T = -\frac{1}{2\pi} e^\phi N^a a_a. \quad (8)$$

The above formula denotes that the four acceleration on the screen is as follows: $a^a = (0, 2\pi T, 0, 0)$, with a local temperature carried by the noncommutative RN screen which is computed in the following form:

$$T = \frac{M_\theta}{2\pi r^2} - \frac{Mr}{4(\pi\theta)^{\frac{3}{2}}} e^{-\frac{r^2}{4\theta}} - \frac{1}{2\pi r^3} \left[Q_\theta^2 + Q^2 \left(\frac{r^2}{2\pi\theta} - \frac{r}{\sqrt{\pi\theta}} \mathcal{E}\left(\frac{r}{2\sqrt{\theta}}\right) \right) e^{-\frac{r^2}{4\theta}} \right]. \quad (9)$$

Note that the local temperature on the event horizon is identical to the Hawking temperature, i.e., $T|_{r=r_H} = T_H$ [7, 12, 17] (see also [18, 19]). In the limit of $\theta \rightarrow 0$, one retrieves the standard temperature for the RN case, i.e.

$$T_{RN} = \frac{M}{2\pi r^2} - \frac{Q^2}{2\pi r^3}. \quad (10)$$

The alteration in entropy for a test particle with mass m at fixed place nearby the screen is equal to

$$\nabla_a S = -2\pi m N_a. \quad (11)$$

Eventually, the modified Newtonian force law as the entropic force in the presence of the noncommutative RN black hole can now be written as

$$F = \sqrt{g^{ab}F_a F_b} = \frac{mM_\theta}{r^2} - \frac{mMr}{2\sqrt{\pi\theta^3}}e^{-\frac{r^2}{4\theta}} - \frac{m}{r^3} \left[Q_\theta^2 + Q^2 \left(\frac{r^2}{2\pi\theta} - \frac{r}{\sqrt{\pi\theta}} \mathcal{E}\left(\frac{r}{2\sqrt{\theta}}\right) \right) e^{-\frac{r^2}{4\theta}} \right], \quad (12)$$

where $F_a = T\nabla_a S = \left(0, \frac{m}{2\sqrt{g_{00}}} \frac{dg_{00}}{dr}, 0, 0\right)$. If we choose the noncommutative Schwarzschild case, i.e. $Q = 0$, then we have

$$F = \frac{mM_\theta}{r^2} - \frac{mMr}{2\sqrt{\pi\theta^3}}e^{-\frac{r^2}{4\theta}}. \quad (13)$$

For the ordinary RN case, the entropic force becomes [20, 21]

$$F_{RN} = 2\pi m T_{RN} = \frac{mM}{r^2} - \frac{mQ^2}{r^3}. \quad (14)$$

The numerical results of the entropic force versus the radius, for several values of $\frac{Q}{\sqrt{\theta}}$, are displayed in Fig. 1. This figure shows that the peak entropic force drops with decreasing the electric charge. In accord with the figure, in the frame of noncommutative geometry inspired model, the entropic force of the black hole becomes larger with the reduction of the screen radius up to the time when it comes near to a highest definite value and afterwards goes down to zero at the minimal nonzero value of the screen radius, r_0 . In fact, on account of coordinate noncommutativity, the black hole entropic force falls down to zero at the remnant size (does not diverge at all), and therefore ceasing to exist the divergence is obvious.

Assuming that the mass of the source becomes larger than the mass of the test particle and is placed at the origin of coordinate, we suppose the energy connected to the source, including noncommutativity effects, is dispersed on a closed screen of constant redshift ϕ . On this surface, N bits of data are accumulated and the holographic data from the source is encrypted as $dN = dA$ [22], where A is the area of the surface. The energy on the noncommutative RN screen, in agreement with the Gauss's theorem, approves thermal equipartition,

$$E = \frac{1}{2} \int_{\Omega} T dN = \frac{1}{4\pi} \int_{\Omega} e^{\phi} \nabla \phi dA. \quad (15)$$

Thus, we have

$$E = M_\theta - \frac{Mr^3}{2\sqrt{\pi\theta^3}}e^{-\frac{r^2}{4\theta}} - \frac{Q_\theta^2}{r} - Q^2 \left(\frac{r}{2\pi\theta} - \frac{\mathcal{E}\left(\frac{r}{2\sqrt{\theta}}\right)}{\sqrt{\pi\theta}} \right) e^{-\frac{r^2}{4\theta}}. \quad (16)$$

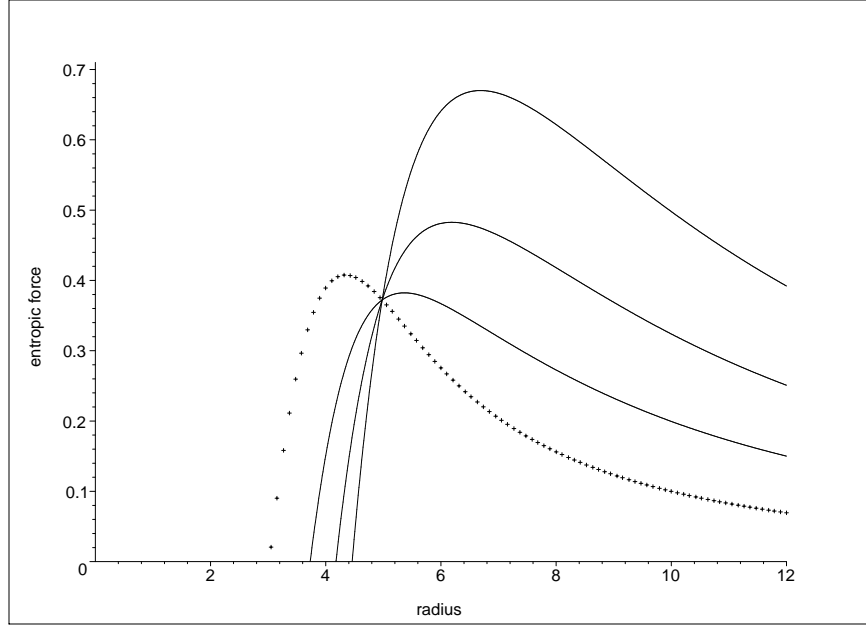


FIG. 1: The entropic force F versus the radius, $\frac{r}{\sqrt{\theta}}$, for some values of $\frac{Q}{\sqrt{\theta}}$. We have set $m = 1.0\sqrt{\theta}$, and $M = 10.0\sqrt{\theta}$. On the right-hand side of the figure, from bottom to top, the solid curves correspond to $Q = 10.0\sqrt{\theta}$, $15.0\sqrt{\theta}$, and $20.0\sqrt{\theta}$, respectively. The cross-dotted curve alludes to the noncommutative Schwarzschild black hole so that it corresponds to $Q = 0$. According to the figure, the appearance of a minimal nonzero radius, r_0 , is evident.

If we consider the case of $Q = 0$, then we can obtain the following relation for the energy on the noncommutative Schwarzschild screen:

$$E = M_\theta - \frac{Mr^3}{2\sqrt{\pi}\theta^3} e^{-\frac{r^2}{4\theta}}. \quad (17)$$

For the commutative case, $\theta \rightarrow 0$, the energy on the ordinary RN screen is as follows [20]:

$$E_{RN} = 2\pi r^2 T_{RN} = M - \frac{Q^2}{r}. \quad (18)$$

The plot presented in Fig. 2 shows the numerical results of the energy versus the radius, for several values of $\frac{Q}{\sqrt{\theta}}$. Fig. 2 clearly shows that for a very large value of the screen radius, $\frac{r}{\sqrt{\theta}} \gg 1$, the energy on the screen will be constant and the constant peak of the energy increases with enlarging $\frac{Q}{\sqrt{\theta}}$. The disappearance of divergence for the energy on the screen, because of the presence of the residual nonzero size of the black hole, can also be demonstrated by decreasing the value of the screen radius. According to the figure, as electric charge becomes larger the minimal nonzero radius increases; this corresponds to Fig. 1.

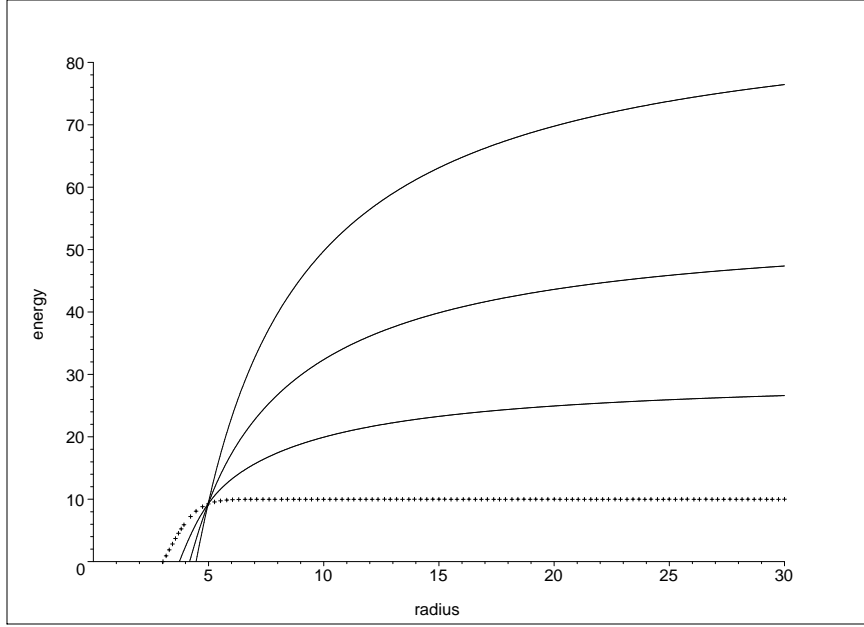


FIG. 2: The energy, $\frac{E}{\sqrt{\theta}}$, versus the radius, $\frac{r}{\sqrt{\theta}}$, for some values of $\frac{Q}{\sqrt{\theta}}$. We have set $m = 1.0\sqrt{\theta}$, and $M = 10.0\sqrt{\theta}$. On the right-hand side of the figure, from bottom to top, the solid curves correspond to $Q = 10.0\sqrt{\theta}$, $15.0\sqrt{\theta}$, and $20.0\sqrt{\theta}$, respectively. The cross-dotted curve alludes to the noncommutative Schwarzschild black hole so that it corresponds to $Q = 0$.

As can be seen from these two figures, the entropic force and the energy on a holographic screen with radius r_0 are zero. This is a prominent consequence which means that since r_0 is the radius of smallest holographic screen, it cannot probe via a test particle which is placed within a short distance from the source. Therefore the standard formulation of Newtonian gravity, in very short distances when the screen radius reaches to r_0 , is broken down. This means that the test particle with mass m cannot identify any gravitational field, when it is at a minimal distance r_0 from the source mass. This phenomenon violates the existence of the mere gravitational interaction for an inert residue of the black hole. The black hole remnant as an indispensable physical object is greatly approved in the quantum gravity literature when quantum gravitational fluctuations are revealed. For instance, when generalized uncertainty principle is taken into account, the complete decay of the black hole into emission is not allowed; then there would be a massive and inert remnant containing the sole gravitational interactions [23]. Our method manifestly demonstrates that the residue of the black hole is entirely inert without any gravitational interaction included. From another point of view, the EP which is relevant to the equality of gravitational

and inertial mass, is contravened because it is now conceivable to distinguish between them. Indeed, the gravitational mass in the leftover size of the inert black hole does not radiate any gravitational field, hence it is identified to be zero as opposed to the inertial mass.

In summary, we have discussed some aspects of Verlinde's proposal in the presence of noncommutative charged black hole based on Gaussian-smeared mass distribution. We have considered the case of noncommutative geometry inspired Reissner-Nordström black hole to improve the expression of black hole's entropic force by taking into consideration the noncommutative corrections. In this setup, we have exhibited that there is a violation of the EP when we apply the noncommutativity effects in our computations, i.e. one can distinguish between a uniform acceleration and a gravitational field. In conclusion, it seems that the noncommutative gravity effects exhibit a conflict with the entropic idea of Newtonian gravity. However, it is possible to assume that both entropic gravity and noncommutative geometry can be correct but in short distances one predicts a violation of EP. This means that it may be feasible that when one approaches r_0 one diverges from GR. Hence, there can be a prediction that EP is contravened at some small scale (maybe the Planck scale) caused by the combination of entropic gravity and noncommutative geometry.

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